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**A real options approach to smoking cessation**

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**When to Quit Under Uncertainty?**  
**A real options approach to smoking cessation**

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**Abstract**

This paper models the decision to quit smoking like an investment decision where the quitter incurs a sunk withdrawal cost today and forgoes their consumer surplus from cigarettes (invests) and hopes to reap an uncertain reward of better health and therefore higher utility in the future (return). We show that a risk-averse mature smoker who expects to benefit from quitting may still rationally choose to delay quitting until they are more confident that quitting is the right decision for them. Such a decision by the smoker is due to the value associated with keeping their option of whether or not to quit open as they learn more about the damage that smoking will have on their future utility. Policies which reduce a smoker's uncertainty about the damage that smoking will have on their future utility is likely to make them quit earlier.

**Keywords:** smoking, quitting, optimal stopping problem, real options analysis, addiction

**JEL Classification:** I1, D1, D8, D9

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## 1. Introduction

Smoking can cause a significant amount of harm to the individual and also inflicts a significant cost to others and the health system (Sloan et al., 2004). In order to better design policies to impact on smoking it is important to understand the underlying reasons for smoking behaviour (Liang et al. 2001). A number of theoretical models have been suggested to explain smoking behaviour. The most discussed is the rational addiction model (Becker and Murphy, 1988) where individuals choose a lifetime consumption path that maximizes their expected lifetime utility taking into account the possibility of addiction. However, the implicit assumption of perfect foresight within this model can impact on the conclusions reached.

Orphanides and Zervos (1995) illustrate theoretically why uncertainty regarding the likelihood of any individual becoming addicted can impact on the consumption decision which helps explain why the young experiment with smoking.<sup>1</sup> Coppejans et al. (2007) empirically examine the impact of price uncertainty on young smokers and find that larger uncertainty reduces both participation and the amount smoked. In the current paper we explore how the uncertainty regarding the damage that continuing to smoke has on health and future utility, impacts on the decision to quit for mature smokers.<sup>2</sup>

The quit decision is often modelled as a decision of whether or not to quit (Becker and Murphy, 1988; Jones, 1999; Douglas, 1998), however, a decision to continue smoking may instead be a decision to delay quitting until sometime in the future.<sup>3</sup> An individual who continues smoking may still have an intention to quit in the future but only when they are more confident that the benefits of quitting in terms of improvements in utility from higher levels of future health outweigh the costs. Amos et al. (2006) find in their qualitative analysis of teenage smokers that most anticipated quitting when older

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<sup>1</sup> Wang (2007) also explores the implications for smoking behaviour when individuals have an uncertain consumption stock threshold above which they become addicted and another uncertain threshold below which they recover from their addiction such that there is a value associated with experimenting to find out these thresholds.

<sup>2</sup> The quit decision is particularly relevant for mature smokers where a “cold turkey” approach to quitting is often seen as the most rational (Becker and Murphy, 1988) and this is supported by empirical evidence from the US where 85% of former smokers who successfully quit stopped suddenly (Sloan and Wang, 2008).

<sup>3</sup> Willemsen (2005) finds that 25% of his Dutch sample of occasional smokers stated that they intended to quit within a year and Sloan et al. (2003) note that for mature smokers their actions suggest a life’s plan that includes continued smoking for “a while longer” before slow or immediate cessation.

from reasons, such as, ill-health, being told to give up by their doctor or simply getting older. Arcidiacono et al. (2007) empirically consider smoking behaviour in the presence of uncertainty regarding health and income for older smokers, however, they do not account for the value associated with delaying the quit decision or how the level of uncertainty impacts on the decision to quit.

Below we show that quitting an addictive habit can be modelled in a similar manner to the decision of whether or not to make an investment given an uncertain future. With the real options approach for investment the investor chooses whether to invest today or delay the investment decision until later given the current economic climate. Once the investment is made there is the sunk cost which cannot be fully recovered so that losses could occur if the economic climate moves in an undesired direction. So there is a value to the investor associated with waiting and keeping the option of whether or not to invest open (see Dixit and Pindyck (1994) for a more detailed discussion).

In the same way an addicted smoker chooses whether to quit today or delay the decision until later when there may be greater certainty that quitting will be beneficial for them. When the smoker quits they incur both a sunk withdrawal cost<sup>4</sup> which cannot be recovered even if at some stage in the future the smoker decides it is worthwhile to start smoking again and also the opportunity cost relating to the lost consumer surplus from forgone cigarette consumption. Thus, if by waiting an individual can learn more about whether or not quitting will be beneficial for them then there is a value associated with continuing to smoke and keeping the option of whether or not to quit open. As noted by Sloan et al. (2003) “timing is everything for mature long-term smokers” therefore understanding the relevance of uncertainty on the timing of their decision to quit is of particular importance for policy makers.

In this paper we examine the decision of whether and when a mature smoker should rationally choose to quit using a real options approach. We show that even for a risk-averse smoker whose expected utility from continuing to smoke is lower than their

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<sup>4</sup> These include both the mental and physical discomforts from quitting which can include; anxiety, restlessness, sleep disturbance, irritability, sweating, difficulty in concentration, listlessness and depression (Ashton and Stepney, 1982). Quitting smoking is difficult with empirical evidence showing that on average it takes 3-4 attempts before an individual successfully quits (Raw et al., 1998). The probability of quitting also varies across individual characteristics, such as education (Kjellsson et al., 2010).

expected utility from quitting may still rationally choose to continue smoking due to the uncertainty involved.

The rest of the paper is organised as follows. Section 2 sets out the basic theoretical model for the decision to quit assuming that the damage from smoking follows a stochastic process represented by geometrical Brownian motion without drift. Section 3 uses simulation methods to explore the impact of changes in key parameters, such as, price, withdrawal costs, risk aversion and the level of uncertainty to the threshold where an individual should decide to quit. Section 4 explores the implications on the quit decision of considering the possibility of a sudden deterioration in health from smoking. Section 5 discusses the potential implications for policy and Section 6 concludes the paper and sets an agenda for future research.

## 2. A Model of Quitting Smoking

We assume that a smoker rationally evaluates whether to quit today.<sup>5</sup> On one hand, he knows that smoking gives additional utility to his daily life (Sloan and Wang, 2008), while on the other hand cigarettes cost him money and *could* damage his long-term health (Kenfield et al., 2008) which will reduce his future utility. He also knows that because of the addictive nature of cigarettes there are some withdrawal costs associated with any attempt to quit (Ashton and Stepney, 1982).

Smoking causes accumulation of addictive and harmful substances in body,  $S$ , which leads to a higher incidence of diseases, such as, coronary heart disease, respiratory disease, chronic obstructive pulmonary disease, lung and others cancers (Kenfield et al., 2008). It is assumed that the human body can deal with some of the harmful substances such that this addictive and harmful stock  $S$  exponentially decays at a rate of  $\delta$ . This is plausible given that after quitting the risk of smoking related diseases decreases over time back to levels consistent with a non-smoker (Doll et al. 2004) and is the same assumption

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<sup>5</sup> While in the current model we assume time consistent preferences, the value of delaying the quit decision would also be applicable in a time-inconsistent setup (for a time-inconsistent setup see Gruber and Koszegi, 2004; Gruber and Koszegi, 2001 and O'Donoghue and Rabin, 1999).

made by others (Becker and Murphy, 1988; Carbone et al., 2005).<sup>6</sup> Therefore, the stock of harmful substances  $S$  for the smoker is denoted by the following process over time:

$$(1) \quad dS_t = [c - \delta S_t] dt$$

where  $c$  is the annual intake of cigarettes which, for simplicity, is assumed constant<sup>7</sup> but will equal zero if and when the smoker quits.

The smoker knows that their smoking is *likely* to damage their health. However, the smoker is uncertain about both the extent to which their smoking *will* damage their future health and their expected utility loss from lower future health (Arcidiacono et al., 2007).<sup>8</sup> The smoking utility damage at time  $t$  is denoted by the following function;

$$(2) \quad \text{damage to health and utility} = g_t^\mu S_t,$$

where  $g_t$  is the damage index which proxies for the health and associated utility damage that each unit of harmful smoking stock will have at time  $t$  and is a source of uncertainty; and  $\mu$  is a positive parameter related to an individual's preferences regarding risk aversion. For a risk-averse individual  $\mu > 1$ , given that the possibility of a higher level of damage will provide even more disutility for the individual.<sup>9</sup>

We assume the smoker can observe the intertemporal damage that their previous smoking has on their health and utility, and therefore a rational individual will try to minimise their intertemporal damage function related to smoking;

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<sup>6</sup> Carbone et al. (2005) assume different depreciation rates for the addictive smoking stock and the harmful smoking stock, although in the current analysis we simplify the analysis by assuming the same depreciation rate for both.

<sup>7</sup> This differs from Becker and Murphy (1988) and Carbone et al. (2005) who allow the intake of cigarettes to vary over time. Assuming a constant rate of consumption at the optimal level allows us to focus our analysis on the optimal stopping problem of when to quit for the mature smoker rather than the optimal amount of cigarettes to smoke.

<sup>8</sup> Some uncertainty is due to people not knowing whether they will have good or bad luck (or genes as the case may be) and there is also uncertainty because individuals do not know the future utility they will be giving up if or when they experience poor health from smoking (they may die prematurely from an unrelated cause, become unemployed or unwillingly divorced at some time in the future which may reduce the impact of the poor health related to smoking on their future lifetime utility).

<sup>9</sup> Note that for this to hold we assume parameters for the model such that  $\bar{g} \geq 1$  and thus once at  $\bar{g}$  the chance that  $g_t$  goes below 1 is very small.

$$(3) \quad V = \int_0^{\infty} g_t^{\mu} S_t e^{-\rho t} dt ,$$

where  $\rho$  is the discount rate that the smoker places on future utility losses.<sup>10</sup> For simplicity we assume that at each point in time, the smoker evaluates the effect of his smoking on health and utility, for example, by observing the differences in health between himself and his non-smoking peers, and thus is able to estimate the current level of his smoking damage index  $g_t$ . He may also observe that the damage/discomfort from past smoking varies over time. However, the damage index  $g_t$  might reach a certain threshold, such that, he would like to quit today. There is extensive evidence that most attempts to quit smoking are made for health reasons (Larabie, 2005; Khwaja et al. 2007) with those who experience a negative health shock being more likely to quit (Sundmacher, 2012; Arcidiacono et al., 2006; Khwaja et al. 2006; Clark and Éttilé, 2002; Clark and Éttilé, 2006; Smith et al., 2001; Hsieh, 1998).

To capture the uncertainty around the damage from the harmful smoking stock on future utility, we assume that the level of  $g_t$  follows a geometrical Brownian motion without drift such that:

$$(4) \quad dg_t = \sigma g_t dW_t ,$$

where  $\sigma$  is the uncertainty parameter and  $W$  is a standard Wiener process. The Brownian motion captures the slow changes in health and utility related to the harmful smoking stock  $S$ . We also explore the implications of a possible sudden deterioration in health and thus utility due to the harmful smoking stock through a Poisson jump process for  $g_t$  in section 4.

### *2.1 The optimal quitting decision for a mature smoker*

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<sup>10</sup> Here we conveniently assume that the smoker lives forever. Such an assumption can be relaxed in a non-perpetual model setup.

Here we consider the decision of when to quit for a mature smoker. In particular for the mature smoker their addictive stock is assumed to be at its steady state, such that,  $S_t = c / \delta$ . The mature smoker faces an optimal stopping problem: to be or not to be a smoker. He could keep smoking as usual, maintain  $S_t$  and continue to incur  $g_t^\mu S_t$  worth of damage, and keep his (real) options to quit later available; postpone the decision of whether to quit or not to sometime in the future when he will have further information about his future levels of  $g_t$ . Alternatively, he could choose to quit now such that he incurs the withdrawal costs today and the level of  $S_t$  declines over time as his body gradually recovers from the damage of the harmful substances. Below equations (5) and (6) set out the intertemporal damage function relating to the choice of continuing to smoke ( $V^N$ ) and quitting ( $V^Q$ ), respectively.

$$(5) \quad \text{Smoking (No Quitting): } V^N = \int_0^\infty g_t^\mu S_t e^{-\rho t} dt, \text{ s. t. } dS_t = [c - \delta S_t] dt \text{ and (4)}$$

$$(6) \quad \text{No Smoking (Quitting): } V^Q = \int_0^\infty g_t^\mu S_t e^{-\rho t} dt, \text{ s. t. } dS_t = -\delta S_t dt \text{ and (4)}$$

For simplicity, we assume that the non-smoking state is credible, such that once the smoker quits they quit forever. As such we ignore the question of whether any particular quitting attempt is successful or the conditions under which a previous smoker will re-start their habit. Though these are interesting and important questions we leave them for future work.<sup>11</sup>

The difference between equations (5) and (6) is the cigarette consumption over time, which implies that in the non-smoking state, the intertemporal utility damage  $V^Q$  will on average decline over time as the level of  $S_t$  declines at a rate of  $\delta$ . The decision by the smoker to quit depends on whether the benefit from quitting in terms of reduced future damage ( $V^N - V^Q$ ) outweighs the withdrawal cost of quitting and the lost consumer surplus from future cigarette consumption ( $K$ ) plus the value of the sacrificed real

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<sup>11</sup> The ability to re-start the habit will reduce the value of waiting though due to the sunk withdrawal costs there will still be a value associated with delaying the quitting decision.



options to instead quit at some stage in the future. The equality of the two states yields a *non-smoking condition*:

$$(7) \quad V^N - V^Q = K + (\text{Value of the sacrificed Real Options to quit later})$$

It is assumed, like Yen and Jones (1996), that the withdrawing discomfort is a positive linear function of the level of addictive stock ( $S$ ). Thus,

$$(8) \quad K = \gamma S + (u_c - p_c) \int_0^\infty c e^{-\rho t} dt = \gamma S + \frac{(u_c - p_c)c}{\rho} \quad \gamma, u_c, p_c > 0$$

where  $\gamma$  is the withdrawal cost per unit of addictive stock,  $p_c$  is price of cigarettes and  $u_c$  is the average utility that the smoker receives from smoking cigarettes.<sup>12</sup> The smoker is likely to receive a consumer surplus from smoking  $(u_c - p_c) > 0$ , however, it may be the case that  $(u_c - p_c) < 0$  if prices are higher than the current utility from smoking as the smoker has not yet quit because the immediate withdrawal costs are too high (which leads to the ‘unhappy addict’ (Suranovic et al., 1999)).

For the optimal stopping problem of quitting we then have the following value-matching condition for the non-smoking condition:

$$(9) \quad V^N(\bar{g}) - V^Q(\bar{g}) = \gamma S + \frac{(u_c - p_c)c}{\rho} + V^R(\bar{g})$$

where  $V^R$  denotes the value of the real options to quit and  $\bar{g}$  denotes the threshold where the smoker should like to quit. Individuals are considering whether to quit, sacrificing their real options to quit later and paying the sunk costs to gain the net marginal benefit due to reductions in the damage from smoking. The smoker quits when the damage index  $g$  is greater than  $\bar{g}$  and the smoker keeps smoking while the damage index  $g$  is

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<sup>12</sup> Here we conveniently assume that the smoker uses the same discount rate as he uses for discounting the health and utility damage function and that prices and future utility from smoking are constant over time.

lower than  $\bar{g}$ . At  $\bar{g}$ , the smoker is indifferent between quitting and not quitting and equation (9) holds.

Here we need to solve  $V^N$ ,  $V^Q$  and the value of the sacrificed real options to quit ( $V^R$ ), to obtain the optimal stopping condition for smoking. The corresponding Bellman equation for  $V^N$  and  $V^Q$  are denoted by the following,

$$(10) \quad \rho V^N = g^\mu S + (c - \delta S) V_s^N + \frac{1}{2} \sigma^2 g^2 V_{gg}^N$$

$$(11) \quad \rho V^Q = g^\mu S - \delta S V_s^Q + \frac{1}{2} \sigma^2 g^2 V_{gg}^Q$$

The solution to equation (10) which denotes that individuals never quit can be computed from the particular solution; and similarly, the solution to equation (11) is a particular solution as the smoker is assumed not to re-start their habit. We show in Appendix A that the particular integrals for  $V^N$  and  $V^Q$  are respectively,

$$(12) \quad V^N(g) = \frac{g^\mu S}{r + \delta} + \frac{c g^\mu}{r(r + \delta)},$$

$$(13) \quad V^Q(g) = \frac{g^\mu S}{r + \delta}.$$

where  $r = \rho - 0.5\mu(\mu - 1)\sigma^2$ . The first term of equation (12) is the same as equation (13) and shows the expected unavoidable discounted damage to utility due to past smoking; and the final term of equation (12) represents the expected discounted damage to utility due to the smoker continuing to smoke over time. Equations (12) and (13) show that the expected net marginal benefit of quitting versus never quitting is thus denoted by

$$(14) \quad V^N - V^Q = \frac{c g^\mu}{r(r + \delta)}.$$

This expected net benefit takes into account all future possibilities for the path of the damage index  $g_t$ . However the expected benefit from quitting today versus not quitting today is less because a decision to continue smoking today does not rule out the possibility to quit at some point in the future. In particular if the damage index moves in an undesirable direction then the smoker still has the option to quit then and thus this limits the potential future damage from not quitting today. Therefore, it is natural to assume that the value associated with having the option to quit later available (the value of the real options to quit) is a function of the same component  $g^\mu$  of this net benefit, or more precisely the Bellman equation for  $V^R$  is,<sup>13</sup>

$$(15) \quad \rho V^R = \frac{1}{2} \sigma^2 g^2 V_{gg}^R.$$

One can easily derive that the solution to equation (15) is as follows,

$$(16) \quad V^R(\bar{g}) = A_1 (\bar{g}^\mu)^{\alpha_1},$$

where  $A_1$  is an unknown positive parameter to be determined by the smooth-pasting condition and  $\alpha_1$  is the positive root of the following characteristic function

$$(17) \quad \frac{1}{2} \sigma^2 \mu \alpha (\mu \alpha - 1) - \rho = 0.$$

Equation (16) denotes the value of the real options foregone due to quitting – once options are exercised, they are gone and thus counted as costs.

As previously mentioned we are considering the quitting decision of a mature smoker. Therefore in equilibrium, the mature smoker would have  $\delta S$  units of harmful

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<sup>13</sup> Note that the term  $(c - \delta S)V_S^R$  is not present in equation (15) because the net benefit of quitting is not a function of  $S$ , the level of harmful substances. Another way is to consider the value of the sacrificed real options to quit,  $V^R$  coming from the homogeneous part of equation (10), but with  $c - \delta S_t = 0$  for the mature smoker.

substances depreciating/absorbed by the body, while he consumes a constant amount of cigarettes  $c$ . Hence, in equilibrium, we can replace  $c$  by  $\delta S$  for  $dS/dt = 0 = c - \delta S_t$ .<sup>14</sup> Thus, the value-matching condition of equation (9) becomes;

$$(18) \quad A_1 (\bar{g}^\mu)^{\alpha_1} + \gamma S + \frac{(u_c - p_c) \delta S}{\rho} = \frac{\delta S \bar{g}^\mu}{r(r + \delta)}$$

where  $\bar{g}$  again denotes the thresholds where the smoker would like to quit. In other words, the smoker quits when the damage index  $g$  is greater than  $\bar{g}$  and the smoker keeps smoking while the damage index  $g$  is lower than  $\bar{g}$ . The smooth-pasting condition follows:

$$(19) \quad \mu \alpha_1 A_1 \bar{g}^{\mu \alpha_1} = \frac{\mu \delta S \bar{g}^{\mu-1}}{r(r + \delta)},$$

which guarantees that the rate of changes with respect to  $g$  are the same before and after  $\bar{g}$ .<sup>15</sup> Equations (18) and (19) give the solutions of the system. Substituting (19) back into the value-matching condition gives the damage index threshold for quitting smoking;

$$(20) \quad \bar{g} = \left[ \left( \frac{\gamma}{\delta} + \frac{(u_c - p_c)}{\rho} \right) \frac{\alpha_1 r (r + \delta)}{(\alpha_1 - 1)} \right]^{\frac{1}{\mu}}$$

Equation (20) clearly indicates that the damage threshold for the smoker to quit is higher when the withdrawal costs are higher, utility derived from smoking is higher and the cost of cigarettes is lower. After obtaining the *damage index threshold* ( $\bar{g}$ ), one can then also obtain the *current utility damage threshold* ( $\bar{D}$ ) which represents the total damage from

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<sup>14</sup> The analysis refers to mature smokers who have already been smoking for a number of years rather than young smokers who have yet to become heavily addicted and are still building up their harmful stocks.

<sup>15</sup> If this was not the case then this would not represent the optimal stopping point (see Dixit and Pindyck, 1994 for a more detailed explanation).

smoking on utility for the individual which also takes into account their level of risk aversion and addictive stock,

$$(21) \quad \bar{D} = S\bar{g}^\mu$$

In the next section we parameterise the model and explore the impacts of changes in various parameters on the threshold for quitting.

### 3. Simulations to explore the impact of changes in key parameters

In order to explore the sensitivity of the quit decision due to changes in the relevant parameters we conduct a number of numerical simulation experiments. As a starting point we assume; a discount rate of 10% ( $\rho = 0.1$ ), harmful substances depreciate at a rate of 17.5% in the body ( $\delta = 0.175$ )<sup>16</sup>, that the individual is risk-averse such that  $\mu = 1.5$  and the damage index has a 10% standard deviation ( $\sigma = 0.1$ ).

We assume that the smoker consumes 1 unit of cigarettes per year and the current utility from smoking ( $u_c = 20$ ) outweighs the price ( $p_c = 10$ ).<sup>17</sup> The level of additive stock,  $S_t$ , for the mature smoker (i.e. in equilibrium) can be derived from equation (1):  $dS_t = [c - \delta S_t]dt$ , such that  $S = c / \delta = 5.7$ . The cost of withdrawing  $\gamma S_t$  is assumed to equal to a year's consumer surplus from smoking, such that,  $\gamma = 10 / S = 1.75$ .<sup>18</sup>

#### 3.1 Simulations Results

The following figures illustrate how the damage threshold where mature smokers should quit changes as adjustments are made to key parameters in the model. Figure 1 illustrates how the uncertainty around the damage index,  $\sigma$ , and the level of risk aversion,

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<sup>16</sup> Compared with continuing smokers, former smokers had a 24% reduction in risk for cardiovascular disease mortality within 2 years of quitting for females. The excess risks for total mortality and both cardiovascular disease and total cancer mortality among female former smokers approached the level of that for never smokers after 10 to 14 years of abstinence (Kawachi et al., 1993). This gives an approximate depreciation of 17.5% for the harmful effects of past smoking.

<sup>17</sup> Note that the average utility derived per cigarette may be expected to be higher for a heavy smoker compared to a light smoker.

<sup>18</sup> Arcidiacono et al. (2007) empirically estimates that the withdrawal costs of quitting are large for mature smokers.

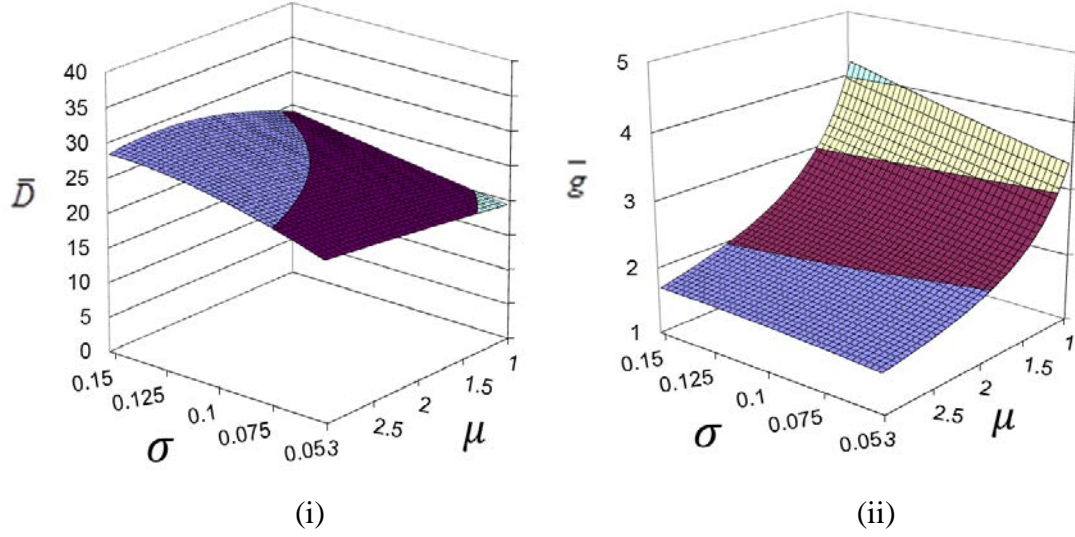
$\mu$ , interact to influence the current utility damage threshold (i) and the damage index threshold (ii) where the smoker should decide to quit.<sup>19</sup> At low levels of risk aversion, the higher uncertainty around the damage index ( $\sigma$ ) the greater is the value of leaving the real options to quit later open and therefore a higher level of damage is required before the smoker should decide to quit. Therefore, less uncertainty around the damage smoking will have on future health and thus utility should make the smoker quit earlier because the individual will be more confident that quitting will be beneficial and little can be learned by delaying the decision.

When the individual is highly risk-averse this pattern changes slightly, while increases in the level of uncertainty from low levels increase the threshold where the smoker should wish to quit, as uncertainty continues to rise the threshold for quitting flattens out. This is due to the two opposing effects relating to uncertainty. First, more uncertainty increases the value of delaying as more can be learned about the damage smoking will have on future utility by waiting and second, for a risk-averse individual more uncertainty increases the benefit of quitting as quitting reduces the individual's exposure to potentially high levels of damage in the future from continuing to smoke. As risk aversion increases both of these effects increase but the second effect increases at a faster rate than the first. Thus, depending on the level of risk aversion, increases in uncertainty can provoke different responses in terms of smoking cessation.

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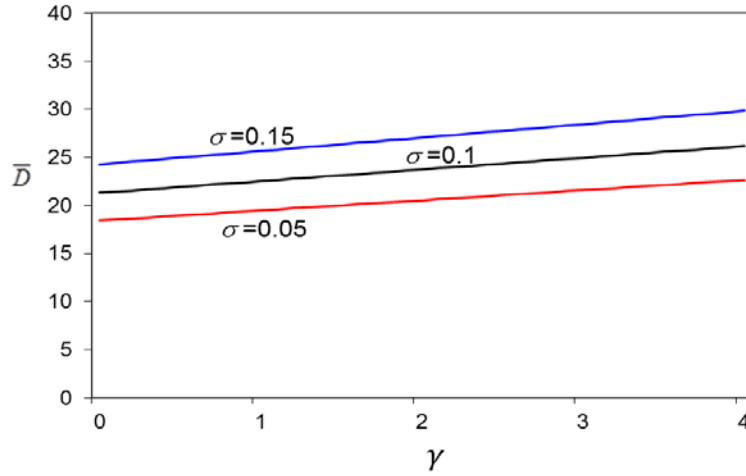
<sup>19</sup> Note that the perceived current level of utility damage  $\bar{D}$  depends on the individual's level of risk aversion.

Figure 1. Impact of uncertainty ( $\sigma$ ) and risk aversion ( $\mu$ ) on the utility damage ( $\bar{D}$ ) and the damage index ( $\bar{g}$ ) quitting thresholds



Note that for the rest of the figures we only illustrate how the current utility damage threshold ( $\bar{D}$ ) changes because the pattern for  $\bar{g}$  is the same given that we hold the level of risk aversion and the equilibrium level of  $S_t$  constant. In Figure 2, as expected, we see the greater the withdrawal costs ( $\gamma$ ) the higher the level of damage is needed before the smoker should quit. Treatments or other cessation programmes, such as, nicotine patches which serve to reduce the withdrawal costs should make smokers more likely to quit earlier.

Figure 2. Impact of the withdrawal cost ( $\gamma$ ) on the quitting threshold ( $\bar{D}$ ) by uncertainty level ( $\sigma$ )

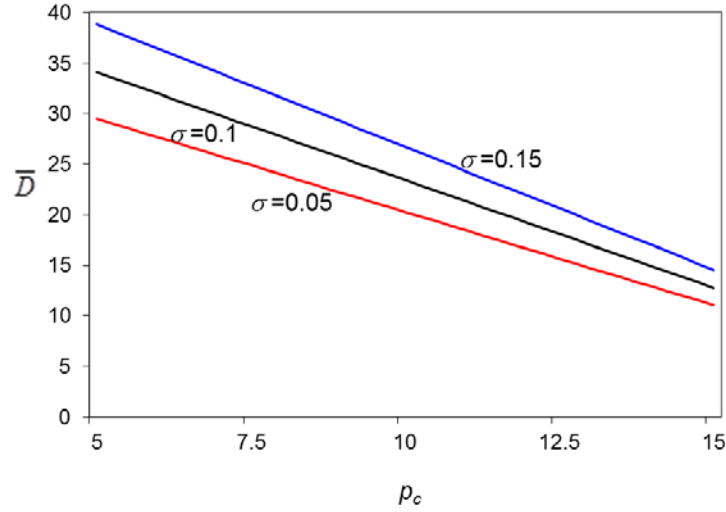


Also as expected, in Figure 3 higher cigarette prices ( $p_c$ ) lead to quitting at a smaller damage threshold as the consumer surplus from continuing to smoke reduces. Furthermore as the price increases the level of uncertainty has a smaller impact on the damage threshold where the smoker should quit. This is because when the potential future consumer surplus from continuing to smoke reduces this in turn reduces the value of the sacrificed real options to quit at some point in the future. It also follows that if the average utility from smoking ( $u$ ) is higher, which may be likely for heavier smokers, then the individual will need a higher level of damage to make them decide to quit (figure not shown).<sup>20</sup>

<sup>20</sup> The average utility from smoking ( $u$ ) enters the theoretical model in the same way as the price ( $p$ ) except with the opposite sign.



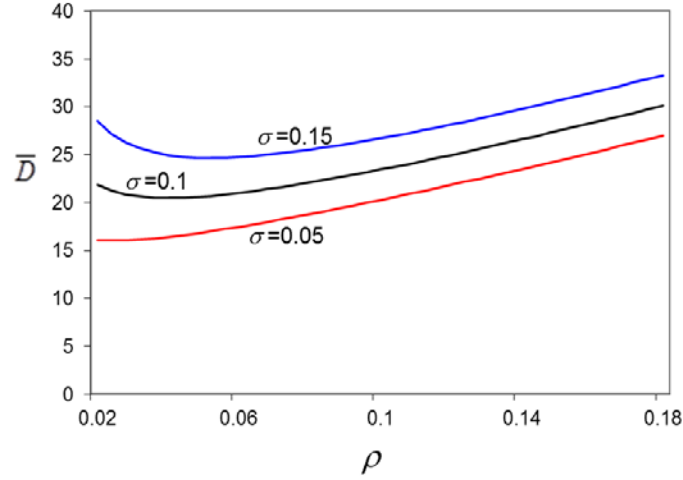
Figure 3. Impact of price ( $p_c$ ) on the quitting threshold ( $\bar{D}$ ) by uncertainty level ( $\sigma$ )



As can be seen in Figure 4, when the discount rate ( $\rho$ ) increases from almost all reasonable levels this results in the smoker only quitting at a higher damage threshold. This is because as the discount rate increases the relative cost of quitting versus the cost of continuing to smoke increases as the withdrawal costs are experienced now while much of the benefit in terms of reduced damage is experienced in the future. However, when the discount rate is extremely low, the value of leaving open the option to quit in the future dominates and thus the smoker is more likely to delay quitting with this effect being more pronounced when there is greater uncertainty. We caution against reading too much into this last result as it is due to a combination of two simplifying assumptions; that the quitter cannot re-start their habit and that they live forever, therefore, the value of the sacrificed real options to quit later is very large at low discount rates as the possible “incorrect” choice of quitting today could be felt forever more.<sup>21</sup>

<sup>21</sup> “Incorrect” here refer to the fact that the damage index may follow a favourable path for the smoker such that their future damage from smoking is actually small.

Figure 4. Impact of the discount rate ( $\rho$ ) on the quitting threshold ( $\bar{D}$ ) by uncertainty level ( $\sigma$ )



#### 4. The impact of a possible sudden deterioration in health due to smoking

In the preceding section we assumed that the damage from the harmful smoking stock changes slowly over time according to Brownian motion, however, smoking may also cause a sudden deterioration in health due to a particular disease event. In this section, we investigate the implications of this on the quitting decision of the mature smoker.

In particular, we assume that there is certain possibility that smoking and its accumulated harmful substances  $S$  would lead some disease event that would make smokers want to quit today.<sup>22</sup> In other words, there is a small possibility that the damage index  $g$  would jump to a certain level  $y_0$ , where  $y_0$  is assumed to be greater than the quitting threshold  $\bar{g}$ . To capture this jump process of the damage from smoking on utility, we now assume that the level of  $g_t$  instead follows a geometrical Brownian motion without drift combined with a Poisson jump, and thus equation (4) becomes

$$(22) \quad dg_t = \sigma g_t dW_t + dJ_t,$$

<sup>22</sup> Many smokers quit following a major adverse health event (Sloan et al., 2003).

where  $\sigma$  is the uncertainty parameter,  $W$  is a standard Wiener process, and  $dJ_t$  is the increment of a Poisson process with a mean event rate  $\lambda$ . It is assumed that if a disease event occurs the level of  $g_t$ , the damage index, increases and jumps to a new much higher point  $y_0$  such that  $y_0 > \bar{g}$ . We assume that following this jump the damage index then continues to fluctuate from this point following the geometrical Brownian motion without drift with the same level of uncertainty as before such that

$$(23) \quad dy_t = \sigma y_t dW_t ,$$

where  $y_t$  is the value the damage index  $g_t$  takes after the jump to  $y_0$ . Note that we assume that once the damage jumps to this new point  $y_0$  there is no longer a possibility of another jump. This Poisson jump captures the fact that there is a chance that a sudden deterioration in health and utility due to the smoking stock may occur, for example, due to a coronary heart disease event.<sup>23</sup>

With the new stochastic process about the damage index  $g_t$ , we can obtain the corresponding Bellman equations and particular solutions for  $V^N$  and  $V^Q$ . It is shown in Appendix B that the benefit from quitting ( $V^N - V^Q$ ) now becomes

$$(24) \quad V^N - V^Q = \frac{cg^\mu}{(r+\lambda)(r+\delta+\lambda)} + \left( \frac{\lambda}{\rho+\lambda} \right) \left( \frac{cy_0^\mu}{r(r+\delta)} + \frac{cy_0^\mu}{(r+\delta)(\rho+\delta+\lambda)} \right),$$

where  $r = \rho - 0.5\mu(\mu-1)\sigma^2$ . Once again, the value of real options to quit later will be a function of the same component as equation (24):  $g^\mu$ . We then have the following value for the real options of the stochastic-jump process (see Appendix C for details),

$$(25) \quad V^R = B_1 g^{\mu\beta_1} + \frac{\lambda}{\rho+\lambda} \left( \frac{cy_0^\mu}{r(r+\delta)} - K \right),$$

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<sup>23</sup> Note, for simplicity, it is assumed that the Wiener process and the Poisson jump are independent.

where  $K = \gamma S + \frac{(u_c - p_c)\delta S}{\rho}$  and  $B_1$  is an unknown positive parameter to be determined

by the smooth-pasting condition  $\beta_1$  is the positive root of the following characteristic function

$$(26) \quad \frac{1}{2}\sigma^2\mu\beta(\mu\beta-1) - (\rho + \lambda) = 0.$$

Thus, the value-matching condition of equation (9) becomes;

$$(27) \quad B_1 \bar{g}^{\mu\beta_1} - \left( \frac{\lambda}{\rho + \lambda} \right) \left( \frac{cy_0^\mu}{(r + \delta)(\rho + \delta + \lambda)} \right) + \frac{\rho}{\rho + \lambda} K = \frac{cg^\mu}{(r + \lambda)(r + \delta + \lambda)},$$

where  $\bar{g}$  is the threshold which makes equation (27) hold. It is easy to see that when  $\lambda$  approaches zero, equation (27) becomes equation (18) – the case without jumps.

The smooth-pasting condition follows:

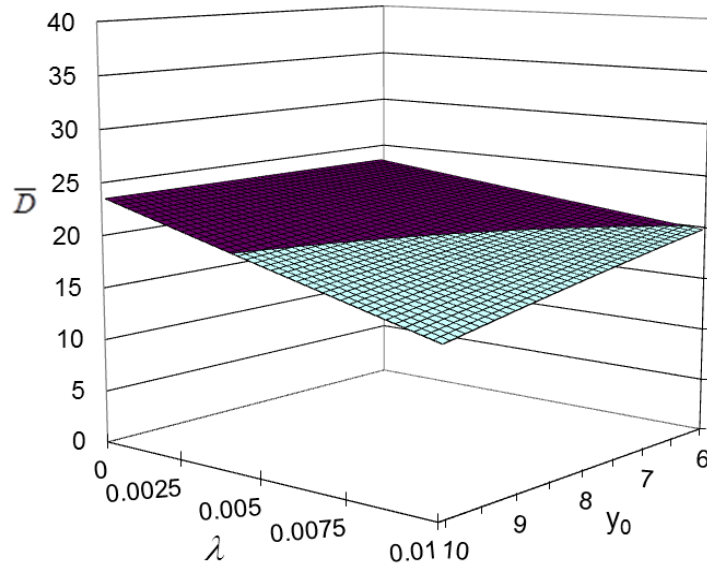
$$(28) \quad \mu\beta_1 B_1 \bar{g}^{\mu\beta_1-1} = \frac{c\mu\bar{g}^{\mu-1}}{(r + \lambda)(r + \delta + \lambda)},$$

which guarantees that the rate of changes with respect to  $g_t$  are the same before and after the quitting threshold. Note that as  $y_0$  is a new higher fixed starting point for the stochastic process  $g$ , the smooth-pasting condition does not depend on the slope with respect to  $y_0$ . Equations (27) and (28) give the solutions of the system.

We again compute the current utility damage threshold ( $\bar{D}$ ) where the smoker should quit and explore how this changes as key parameters change in the model. Here it is assumed that the health event rate for the Poisson jump process,  $\lambda$ , is 0.5% and when the health event occurs, the value of  $g_t$  increases and jumps to  $y_0 = 7 > \bar{g}$ . The rest of parameters are the same as in the previous section.

Figure 5 examines how the rate at which sudden smoking health event occurs  $\lambda$  and the size of this event  $y_0$  impact on the threshold where the smoker should quit. It can be seen that as both the event rate and the size increase, the threshold where the smoker should quit decreases but not as dramatically as one may at first expect. Increases in both the rate of occurrence and the size of this effect increase the expected benefits from quitting which makes smokers want to quit earlier but this is partially offset by increases in the value of the sacrificed real options from quitting due to the increases in uncertainty around the expected benefits from the possible jump.

Figure 5. The impact of a potential sudden jump in the damage index; event rate ( $\lambda$ ) and  $y_0 > \bar{g}$  impact on the quitting threshold ( $\bar{D}$ )



#### 4. Policy implications

From the analysis above we see that for most individuals if they were able to more accurately predict the damage that their smoking would have on their future health and utility (less uncertainty) then we would expect them to quit sooner as they would have less to gain from waiting and learning about the future damage from smoking.<sup>24</sup> Policies

<sup>24</sup> Note that for highly risk-averse individuals being able to better predict the impact of their smoking on their future health may in fact make them more likely to continue smoking.

could reduce the uncertainty by providing smokers with more information about the damage, and expected damage, smoking is likely to be having on their health, for example, advice from doctors which would reduce the value to them by delaying the decision to quit. As expected, subsidizing products which reduce withdrawal costs and increasing the price of cigarette are also likely to encourage smokers to quit.

It should also be noted that the perceived negative impact of continuing to smoke on future utility is likely to be a combination of both the future smoking-related health impacts and how these health impacts affect utility derived from the future consumption of goods. If an individual expects to have low levels of consumption/utility in the future then the damage to their future utility from decreases in future health may also be small. On the other hand, if an individual expects future utility to rise then decreases in future health may have a large impact on lifetime utility. Thus the uncertainty around the damage from smoking is also likely to be a combination of the uncertainty around these two aspects. Barnes and Smith (2009) find that future economic insecurity (future unemployment rates) has a large and statistically significant positive effect on the decision to continue smoking. While they suggest this may in part be due to a self-medicating response to economic insecurity it is also consistent with our model above. When individuals are uncertain about whether they are likely to become unemployed they may be more likely to continue smoking and wait to see whether they do become unemployed; if an individual does become unemployed the “utility loss” from poor future health as a result of smoking may decrease. Thus policies which encourage smokers to plan for the future<sup>25</sup> or policies which produce a more stable society such that smokers were more certain in terms of what they would be giving up from poor future health may also make them quit earlier.

## 5. Conclusion

Given that the timing of the decision to quit by mature smokers is of critical importance it is relevant to consider how the uncertainty around the damage from smoking impacts on this decision. In the current analysis we have demonstrated the

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<sup>25</sup> Khwaja et al. (2007a) find that current smokers have shorter planning horizons than both former smokers and those who never smoked.

usefulness to consider the decision to quit smoking using a real option approach. In doing so we have made a number of assumptions to simplify the analysis, such as, assuming the quitting decision is “credible” (smokers do not start smoking again once they quit), assuming smokers consume a constant amount of cigarettes, that smokers live forever and that the damage from smoking follows particular stochastic processes. These assumptions could be relaxed or modified in future work to provide a more detailed and robust analysis of the decision to quit, however, they do not detract from the key results found in the current paper.

This paper has shown that a risk-averse mature smoker who expects that their utility from quitting today will be greater than their utility from never quitting may still rationally choose to continue smoking due to the uncertainty involved. From the smokers’ perspective not quitting may be an optimal decision as they wait to learn more about the likely impact that their smoking will have on their future utility. We see that for a risk averse individual increases in the uncertainty around the damage smoking has on utility can lead to different effects on the threshold where the smoker should quit depending on the level of risk aversion. While increases in uncertainty increase the benefit of quitting as it increases the risk of continuing to smoke it also increases the value of waiting and therefore the balance of these two effects will determine whether increases in uncertainty increase or decrease the threshold where the smoker should quit. Therefore how smokers perceive the uncertainty around the future damage that smoking will have on them and what they may learn by waiting will have a strong influence on their decision to quit.

Arcidiacono et al. (2007) suggest that much of the uncertainty about the link between smoking and health outcomes is revealed (uncertainly falls) during later years in life which may suggest why quitting rates are higher in some older age group. So far we have ignored the issue of death and therefore our analysis is unlikely to be applicable to those groups where the risk of death in the short term is high. In particular, there may exist an upper damage threshold where the smoker would no longer quit as the damage has already been done and there may be little to be gained from quitting at this late stage and only a lot to be lost in terms of the immediate withdrawal costs.

Analysing the quit decision using a real options approach also provides more support for the stylised facts about addicts' behaviour that may at first seem irrational. In particular, the analysis can help explain why a lot more smokers state that they intend to quit (because the expected benefits outweigh the costs) than those that do attempt to quit (because some of those who intend to quit continue to delay the action until later). In addition, it can also help explain a smoker's regret from not quitting earlier. If the damage experienced by a smoker is what they had expected then they may have preferred to quit earlier had there been no uncertainty around this outcome but instead the uncertainty may have made them delay until later. This is a stronger type of regret than others have justified where individuals regret that life did not turn out how they expected (Orphanides and Zervos, 1995).

The question that still arises is whether a reluctance to quit today is simply due to a personal rational decision, as we further explain in the current paper, or a lack of self-control such that individuals weight present utility more heavily than future utility (Gruber and Koszegi, 2004; Gruber and Koszegi, 2001; O'Donoghue and Rabin, 1999) or perhaps more likely some combination of these two driving forces.<sup>26</sup> Using a real options approach is not only applicable for smoking but also for other addictive habits in which there are sunk withdrawal costs associated with quitting and the future damage is uncertain such that delaying provides further information about whether or not quitting will be beneficial for the individual. Further research is needed to explore the implications for smoking cessation behaviour from a more detailed analysis relaxing some of the assumptions within our current model.

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<sup>26</sup> Even in a time inconsistent framework delaying the decision has value when there is uncertainty regarding the damage of smoking on future health.



## References

- Amos, A., Wiltshire, S., Haw, S. and McNeill, A., 2006, Ambivalence and uncertainty: experiences of and attitudes towards addiction and smoking cessation in the mid-to-late teens, *Health Education Research*, 21, 181-191.
- Arcidiacono P., Sieg H. and Sloan F., 2007. Living rationally under the volcano? An empirical analysis of heavy drinking and smoking. *International Economic Review*, 48, 37-65.
- Ashton, H. and Stepney, R., 1982. *Smoking, Psychology and Pharmacology*. Tavistock Publications, London.
- Barnes, M. and Smith, T., 2009. Tobacco use as response to economic insecurity: Evidence from the National Longitudinal Survey of Youth, *The B.E. Journal of Economic Analysis & Policy*, 9, 47.
- Becker, G.S. and Murphy, K.M., 1988. A theory of rational addiction. *Journal of Political Economy*, 96, 675-700.
- Carbone, J., Kverndokk, S. and Røgeberg, O., 2005. Smoking, health, risk and perception. *Journal of Health Economics*, 24, 631-653.
- Coppejans, M., Gilleskie, D., Sieg, H. and Strumpf, K., 2007. Consumer demand under price uncertainty: Empirical evidence from the market for cigarettes. *Review of Economics and Statistics*, 89, 510-521.
- Clark, A.E. and Étilé, F, 2002. Do health changes affect smoking? Evidence from British panel data, *Journal of Health Economics*, 21, 533-562.
- Clark, A.E. and Étilé, F, 2006. Health changes and smoking: An economic analysis, *Substance Use & Misuse*, 41, 427-451.
- Dixit, A. and Pindyck, R., 1994. *Investment Under Uncertainty*. Princeton: Princeton University Press.
- Doll, R., Peto, R., Boreham, J. and Sutherland, I., 2004. Mortality in relation to smoking: 50 years' observations on male British doctors. *BMJ*, 328: 1519.
- Douglas, S., 1998. The duration of the smoking habit, *Economic Inquiry*, 36, 49-64.
- Gruber, J. and Koszegi, B. 2001. Is addiction rational: Theory and evidence. *Quarterly Journal of Economics*, 116, 1261-1304.
- Gruber, J. And Koszegi, B. 2004. Tax incidence when individuals are time-inconsistent: The case of cigarette excise taxes. *Journal of Public Economics*, 88, 1959-1987.

- Hsieh, C-R., 1998, Health risk and the decision to quit smoking, *Applied Economics*, 30, 795-804.
- Jones, A., 1999. Adjustment costs, withdrawal effects, and cigarette addiction. *Journal of Health Economics*, 18, 125–137.
- Kawachi, I., Colditz, G., Stampfer, M, Willett, W., Manson, J., Rosner, B., Hunter, D., Hennekens, C. and Speizer, F., 1993. Smoking cessation in relation to total mortality rates in women. *Annals of Internal Medicine*, 119, 992-1000.
- Kenfield, S., Stampfer, M., Rosner, B. and Colditz, G., 2008. Smoking and smoking cessation in relation to mortality in women. *JAMA*; 299, 2037-47.
- Khwaja, A., Silverman, D. and Sloan, F., 2007a. Time preference, time discounting and smoking decisions. *Journal of Health Economics*, 26 , 927–949.
- Khwaja, A., Silverman, D., Sloan, F. and Wang, Y. 2007b. Smoking, wealth accumulation and the propensity to plan. *Economic Letters*, 94, 96-103.
- Khwaja A., Sloan F. A., Chung S. 2006. Learning about individual risk and the decision to smoke. *International Journal of Industrial Organisation*, 24, 683–99.
- Kjellsson, G., Gerdtham, U-G. and Lyttkens, CH. 2010. Breaking bad habits by education – Smoking dynamics among Swedish women, *Health Economics*, 20, 876-881.
- Larabie, L. 2005. To what extent do smokers plan quit attempts? *Tobacco Control*, 14, 425–428.
- Liang, L., Chaloupka, F., Nichter, M. and Clayton, R., 2001, Price policies and youth smoking. *Addiction*, 98, 105–22.
- O'Donoghue, T. and Rabin, M. 1999. Doing it now or later. *American Economic Review*, 89, 103-124.
- Orphanides, A. and Zervos, D., 1995. Rational addiction with learning and regret. *Journal of Political Economy*, 103, 739–758.
- Raw, M., McNeill, A. and West, R., 1998. Smoking cessation guidelines for health professionals: A guide to effective smoking cessation interventions for the health care system. *Thorax*. 53, S1–S19.
- Sundmacher, L., 2012. The effect of health shocks on smoking and obesity. *European Journal of Health Economics*, 13, 451-460.
- Sloan, F., Osterman, J., Picone, G., Conover, C. and Taylor D., 2004. *The Price of Smoking*, MIT Press, Cambridge, MA.

Sloan, F., Smith, V. and Taylor, D. 2003. *The smoking puzzle: Information, risk perception and choice*. Cambridge, M A: Havard University Press.

Sloan FA, Wang Y. 2008. Economic theory and evidence on smoking behavior of adults. *Addiction*, 103, 1777-1785.

Smith, V.K., Taylor, D.H., Sloan, F.A., Johnson, F.R., Desvousges, W.H., 2001. Do smokers respond to health shocks? *Review of Economics and Statistics*, 83, 675–687.

Suranovic, S., Goldfarb, R., and Leonard, T. 1999. An economic theory of cigarette addiction, *Journal of Health Economics*, 18, 1-29.

Wang, R. 2007. The optimal consumption and the quitting of harmful addictive goods, *The B.E. Journal of Economic Analysis & Policy*: 7(1), Article 15. Available at: <http://www.bepress.com/bejeap/vol7/iss1/art15>

Willemsen, M. C. (2005). The new EU cigarette health warnings benefit smokers who want to quit the habit: Results from the Dutch continuous survey of smoking habits. *European Journal of Public Health*, 15, 389–392.

Yen, S.T. and Jones, A.M., 1996. Individual cigarette consumption and addiction: a flexible limited dependent variable approach, *Health Economics*, 5 , 105–117.

## Appendix A: Derivation of Equations (12) and (13)

With the result by Ito's Lemma, we have

$$(A1) \quad E[g^\mu] = \frac{1}{2}\sigma^2\mu(\mu-1).$$

The quitting decision where  $c=0$  lead to the depreciation of harmful substances at a rate of  $\delta$ . Therefore, the particular integral for quitting decision is as follows,

$$(A2) \quad V^Q(g) = E\left[\int_0^\infty g_t^\mu S_t e^{-\rho t} dt \middle| S = S_0; g = g_0\right] \\ = \int_0^\infty g^\mu e^{+\frac{1}{2}\mu(\mu-1)\sigma^2 t} S e^{-\delta t} e^{-\rho t} dt = \frac{g^\mu S}{\rho + \delta - \frac{1}{2}\mu(\mu-1)\sigma^2}.$$

which is the equation (13) in the text. Note that it can be easily shown that equation (A2) satisfies equation (11).

The analytical solution to equation (10) is denoted by the following equation,

$$(A3) \quad S_t = S_0 e^{-\delta t} + \frac{c}{\delta} e^{-\delta t} - \frac{c}{\delta}.$$

Substituting equations (A1) and (A3) into the particular integral for no-quitting decision gives

$$(A4) \quad V^N(g) = \int_0^\infty g^\mu e^{+\frac{1}{2}\mu(\mu-1)\sigma^2 t} \left( S e^{-\delta t} - \frac{c}{\delta} e^{-\delta t} + \frac{c}{\delta} \right) e^{-\rho t} dt \\ = \frac{g^\mu S}{\rho + \delta - \frac{1}{2}\mu(\mu-1)\sigma^2} - \frac{c g^\mu}{\delta \left( \rho + \delta - \frac{1}{2}\mu(\mu-1)\sigma^2 \right)} + \frac{c g^\mu}{\delta \left( \rho - \frac{1}{2}\mu(\mu-1)\sigma^2 \right)}.$$

Collecting the last two terms of the right-hand side of equation (A4) gives

$$(A5) \quad V^N(g) = \frac{g^\mu S}{\rho + \delta - \frac{1}{2}\mu(\mu-1)\sigma^2} + \frac{c g^\mu}{\left( \rho - \frac{1}{2}\mu(\mu-1)\sigma^2 \right) \left( \rho + \delta - \frac{1}{2}\mu(\mu-1)\sigma^2 \right)},$$

which is a particular solution to equation (10) in the text and thus equation (A5) concludes the proof of equation (12) in the text.

## Appendix B: Derivation of Equation (24)

With the stochastic processes, equations (22) and (23), we have the following Bellman equations for the decision of no-quitting,

$$(B1) \quad \rho V^N(g, S) = g^\mu S + (c - \delta S) V_S^N(g, S) + \frac{1}{2}\sigma^2 g^2 V_{gg}^N(g, S) + \lambda (V^N(y, S) - V^N(g, S)),$$

$$(B2) \quad \rho V^N(y, S) = g^\mu S + (c - \delta S) V_S^N(y, S) + \frac{1}{2}\sigma^2 g^2 V_{gg}^N(y, S),$$

and decision of quitting,

$$(B3) \quad \rho V^Q(g) = g^\mu S - \delta S V_S^Q(g) + \frac{1}{2}\sigma^2 g^2 V_{gg}^Q(g) + \lambda (V^Q(y, S) - V^Q(g, S)),$$

$$(B4) \quad \rho V^Q(y, S) = g^\mu S - \delta S V_S^Q(y, S) + \frac{1}{2} \sigma^2 g^2 V_{gg}^Q(y, S).$$

Note that there are no jumps in equations (B2) and (B4) – once the jump/certain illness happens, then it would no longer jump to the same illness. Using the same procedure in Appendix A, we obtain the following solutions to (B2) and (B4),

$$(B5) \quad V^N(y, S) = \int_0^\infty y^\mu e^{+\frac{1}{2}\mu(\mu-1)\sigma_h^2 t} \left( S e^{-\delta t} - \frac{c}{\delta} e^{-\delta t} + \frac{c}{\delta} \right) e^{-\rho t} dt = \frac{y_0^\mu S}{r + \delta} + \frac{c y_0^\mu}{r(r + \delta)},$$

$$(B6) \quad V^Q(y, S) = \int_0^\infty y^\mu e^{+\frac{1}{2}\mu(\mu-1)\sigma_h^2 t} (S e^{-\delta t}) e^{-\rho t} dt = \frac{y_0^\mu S}{r + \delta},$$

where  $r = \rho - 0.5\mu(\mu-1)\sigma^2 > 0$ . Substituting equations (B5) and (B6) into equations (B1) and (B3) respectively gives

$$(B7) \quad (\rho + \lambda) V^N(g, S) = g^\mu S + (c - \delta S) V_S^N(g, S) + \frac{1}{2} \sigma^2 g^2 V_{gg}^N(g, S) + \lambda \left( \frac{y_0^\mu S}{r + \delta} + \frac{c y_0^\mu}{r(r + \delta)} \right),$$

$$(B8) \quad (\rho + \lambda) V^Q(g, S) = g^\mu S - \delta S V_S^Q(g, S) + \frac{1}{2} \sigma^2 g^2 V_{gg}^Q(g, S) + \lambda \frac{y_0^\mu S}{r + \delta}.$$

We can assume that  $V^N(g, S)$  has the following solutions

$$(B9) \quad V^N(g, S) = a g^\mu S + b g^\mu + d S + e.$$

Substituting equation (B9) back to (B7) gives

$$(B10) \quad (\rho + \lambda) (a g^\mu S + b g^\mu + d S + e) = g^\mu S + (c - \delta S) (a g^\mu + d) + \frac{1}{2} \sigma^2 (a \mu(\mu-1) g^\mu S + b \mu(\mu-1) g^\mu) + \lambda \left( \frac{y_0^\mu S}{r + \delta} + \frac{c y_0^\mu}{r(r + \delta)} \right).$$

Collecting terms gives

$$(B11) \quad \left[ \left( \rho + \delta + \lambda - \frac{1}{2} \sigma^2 \mu(\mu-1) \right) a - 1 \right] g^\mu S + \left[ \left( \rho + \lambda - \frac{1}{2} \sigma^2 \mu(\mu-1) \right) b - a c \right] g^\mu + \left[ \left( \rho + \delta + \lambda \right) d - \frac{\lambda y_0^\mu S}{r + \delta} \right] S + \left[ \left( \rho + \lambda \right) e - d c - \frac{\lambda c y_0^\mu}{r(r + \delta)} \right] = 0.$$

Equation (B11) holds if each of the brackets on the left-hand side is set to zero. Thus, we have

$$(B12) \quad a = \frac{1}{r + \delta + \lambda}$$

$$(B13) \quad b = \frac{ac}{r + \lambda} = \frac{c}{(r + \lambda)(r + \delta + \lambda)}$$

$$(B14) \quad d = \frac{\lambda y_0^\mu}{(r + \delta)(\rho + \delta + \lambda)}$$

$$(B15) \quad e = \frac{\lambda c y_0^\mu}{(\rho + \lambda)(r + \delta)(\rho + \delta + \lambda)} + \frac{\lambda c y_0^\mu}{r(\rho + \lambda)(r + \delta)}$$

Thus, equation (B9) becomes

$$(B16) \quad V^N(g, S) = \frac{g^\mu S}{r + \delta + \lambda} + \frac{cg^\mu}{(r + \lambda)(r + \delta + \lambda)} + \frac{\lambda y_0^\mu S}{(r + \delta)(\rho + \delta + \lambda)} + \frac{\lambda cy_0^\mu}{(\rho + \lambda)(r + \delta)(\rho + \delta + \lambda)} + \frac{\lambda cy_0^\mu}{r(\rho + \lambda)(r + \delta)}.$$

Similarly, we can obtain the solutions to equation (B8)

$$(B17) \quad V^Q(g, S) = \frac{g^\mu S}{r + \lambda + \delta} + \frac{\lambda y_0^\mu S}{(r + \delta)(\rho + \delta + \lambda)},$$

We then have

$$(B11) \quad V^N - V^Q = \frac{cg^\mu}{(r + \lambda)(r + \delta + \lambda)} + \left( \frac{\lambda}{\rho + \lambda} \right) \left( \frac{cy_0^\mu}{r(r + \delta)} + \frac{cy_0^\mu}{(r + \delta)(\rho + \delta + \lambda)} \right),$$

which is equation (24) in the text.

### Appendix C: Derivation of Equation (25)

The corresponding Bellman equation for stochastic process with jump of equation (22) in the text is denoted by the following differential equation,

$$(C1) \quad \rho V^R(g) = \frac{1}{2} \sigma^2 g^2 V_{gg}^R(g) + \lambda (V^R(y) - V^R(g)),$$

as real options are not a function of  $S$ . Alternatively one can think that for the mature smoker,  $S - \delta c = 0$ , thus  $(S - \delta c)V_S^R = 0$ . As any jump would lead to immediate action of a smoker to quit and exercise the real options to gain the payoff of  $V^N(y) - V^Q(y) - K$ , where  $V^N - V^Q$  follows equation (B5) and (B6). Thus, equation (C1) becomes

$$(C2) \quad \rho V^R(g) = \frac{1}{2} \sigma^2 g^2 V_{gg}^R(g) + \lambda \left( \frac{cy_0^\mu}{r(r + \delta)} - K - V^R(g) \right)$$

Assume that  $V^R$  has the following functional form,

$$(C3) \quad V^R = Bg^{\mu\beta} + E,$$

where  $B$  is an unknown parameter. We then have the following relationship,

$$(C4) \quad \frac{1}{2} \sigma^2 g^2 V_{gg}^R = \frac{1}{2} \sigma^2 \mu\beta(\mu\beta - 1) Bg^{\mu\beta}$$

Substituting equations (C3) and (C4) into (C2) and re-arranging give

$$(C5) \quad B(g^\mu)^\beta \left[ \left( \rho + \lambda - \frac{1}{2} \sigma^2 \mu\beta(\mu\beta - 1) \right) \right] + \left[ E(\rho + \lambda) - \lambda \left( \frac{cy_0^\mu}{r(r + \delta)} - K \right) \right] = 0.$$

Equation (C5) holds if each of the brackets on the left-hand side is set to zero. Thus, we have

$$(C6) \quad \frac{1}{2} \sigma^2 \mu\beta(\mu\beta - 1) - \rho - \lambda = 0$$

$$(C7) \quad E = \frac{\lambda}{\rho + \lambda} \left( \frac{cy_0^\mu}{r(r + \delta)} - K \right)$$

There are two characteristic roots for (C6):  $\beta_1 > 0$  and  $\beta_2 < 0$ . As we do not consider the options to resume smoking, we can ignore the answer related to  $\beta_2 < 0$  as  $\lim_{g \rightarrow 0} A_2 g^{\mu\beta_2} = \infty$ .

Thus, equation (C3) becomes

$$(C8) \quad V^R = B_1 g^{\mu\beta_1} + \frac{\lambda}{\rho + \lambda} \left( \frac{c y_0^\mu}{r(r + \delta)} - K \right),$$

which concludes the proof of equation (25) in the text.